PROPAGATION OF CLEAVAGE CRACKS IN SINGLE CRYSTALS OF LITHIUM FLUORIDE

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Some results of comparing the predictions of the theory of quasibrittle fracture relative to the behavior of cracks with the behavior of cracks in a real material are presented. Predictions of the theory are based on the concept that a crack separates the specimen into two elastic beams rigidly fixed in the crack tip, the same specific energy always being spent during the creation of a new surface. In the investigation we checked whether the constant T figuring in the equations of the theory is constant under the given experimental conditions.

1. The specimens were picked out of LiF single crystals and represented rectangular parallelepipeds with dimensions of about 4×4× 50 mm. The crack developed along the cleavage plane parallel to the long sides of the specimen. The specimens were prepared as in [1].

To record the propagation of the crack we used an SKS-1M motion-picture camera operating in a "streak" regime [2]. The light from an incandescent bulb fell on the plane of the crack and, being reflected from it as from a mirror, struck the objective. The image of the crack on the film represented a narrow shining streak perpendicular to the direction of film travel. As a result of photographing on film we obtained the crack length as a function of time (Fig. 1).

Propagation of the crack was accomplished by two methods:

1) spreading the sides of the crack in the end section of the specimen at constant rates of 195, 550, and 2000 mm/sec;

2) loading the sides of the crack in the end section with free-falling weights. In the first method of loading a pendulum with a mass of several kilograms falling from a certain height, after triggering the camera, struck the specimen, forcing it to move onto a steel wedge whose point was inserted into a preliminarily prepared crack. At the moment of contact of the pendulum and specimen the triggering circuit of an ÉV-1 flashlamp was closed, the light from which left on the film a streak running along the image of the crack. This streak allowed determining the start of movement of the specimen (Fig. 1). Knowing the travel speed of the film, angle of the wedge, and speed of the pendulum, we could determine at any moment the time that passed from the start of movement of the specimen and the distance between the sides of the crack. The diagram of loading in the second case is shown in Fig. 2. The falling weight 1 triggers the camera and then releases carriage 2 with a weight which lies on carriage 3 preliminarily suspended to the sides of the crack on loops of thin copper foil. With a sufficient weight of the load the crack begins to propagate.

2. The basis of the theory of quasibrittle fracture is the assumption of constancy of energy T spent in crack propagation for the creation of a new surface of unit area. This assumption was checked experimentally in the investigation. The relationship of the longitudinal and transverse dimensions of the specimen enabled us to use the beam approximation of crack theory with sufficient grounds.

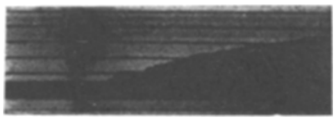

Fig. 1

If the distance 2h between opposite sides of an equilibrium crack in the end section, where the bending moment is absent, is known and

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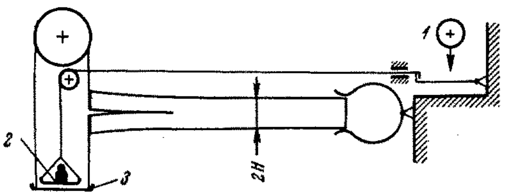

Fig. 2

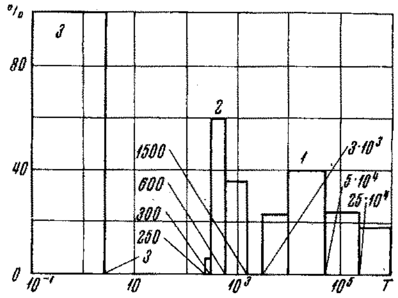

Fig. 3

the crack divides the specimen into two parts with thicknesses H_1 and H_2, the density of the surface energy T is given by the formula

$$T = \frac{3Eh^2}{4l^4} \frac{H_1^3 H_2^3}{H_1^3 + H_2^3} \tag{2.1}$$

where E is Young's modulus and l is the length of the crack. Equation (2.1) is obtained from the condition of the minimum sum of the bending energy and surface energy of the two sides of the crack, whereby both "halves" of the specimen into which the crack divides the original specimen are considered elastic beams rigidly fixed at the crack tip. In deriving (2.1) it is necessary to bear in mind that the shearing forces in the end section of the specimen are equal in both beams. This equality, following from the variational principle as a natural boundary condition, can be obtained directly from an examination of the equilibrium of a wedge spreading the sides of the crack.

If we assign the shearing force mg in the end section, then (provided $H_1 = H_2 = H$)

$$T = \frac{6\,(mgl)^2}{Eb^2H^3} \tag{2.2}$$

where b is the transverse dimension of the specimen parallel to the plane of the crack. The equilibrium determined by Eq. (2.2) is unstable for a fixed weight of the load mg. If the quantity mgl is greater than the "critical" value, the crack begins to propagate, its velocity not exceeding the quantity

$$v = g^{1/2}\,(3EH^3 / 32T)^{1/4} \tag{2.3}$$

where g is the acceleration of gravity (see [3], Sec. 1).

3. The results of treating the experiments are given in Fig. 3. The quantity T is plotted in a semi-logarithmic scale on the x axis, and the distributions 1, 2, and 3 were obtained by Eqs. (2.1)-(2.3), respectively. The height of each rectangle shows what portion of the measurements gave values of T lying within the limits occupied by its base. Distribution 1 was constructed for 50 specimens on which we calculated 165 values of T. This is related with the fact that with loading by the first method the crack often propagates in steps. The crack was considered to be in equilibrium if it remained at rest for 0.0005 sec or longer. We did not find a dependence of T on the rate of spreading of the sides of the cracks under these experimental conditions. The error of determining T by Eq. (2.1) was about 40%. Distributions 2 and 3 were constructed for 50 specimens. With loading by the second method it was necessary to guess beforehand the critical load. Therefore, the force actually applied was either less than the critical if a crack did not develop after loading, or greater than the critical in the opposite case. As a consequence of this inaccuracy the error related with asymmetry in the position of the crack was not taken into account in calculations by Eqs. (2.2) and (2.3). Therefore it is impossible to indicate the a priori error in determining T by (2.2) and (2.3).

As we see from Fig. 3, calculations by the equations given in the preceding section give a considerable divergence in the values of T. The values of T in the case where the distance between sides of the crack are given exceeds by a factor of 1-3 orders the values of T for a given shearing force, although the state of stress is the same in both cases. The value of T in distribution 3 is less by two orders than the value of surface tension (about 700 dyn/cm according to [4]). These facts force us to acknowledge that a real mechanical system is not described truly by the theoretical model (an elastic rigidly fixed beam) on whose basis the calculating formulas were derived.

The divergence of the values of T in distributions 1 and 2 indicates directly that a real system is more yielding than follows from the theoretical model. Therefore, to explain the experimental results we must provide the model of the system being considered with an additional degree of freedom which could accumulate potential energy as long as the crack is stationary and release it when the crack propagates.

The latter property should explain the extremely large values of the crack velocity in loading by the second method, i.e., the extremely small values of T in distribution 3.

We will reject, for example, that condition of rigid fixing which forbids turning at the crack tip and postulate the existence in the fixed support of a linearly elastic hinge whose angle of turn is proportional to the bending moment in the support. Formally, this will be expressed in that a term proportional to the square of the angle of turn and equal to the energy stored in the hinge will be added to the potential energy of the system.

To validate this replacement in the boundary conditions, we mention that the conditions of rigid fixing introduced into crack theory by I. V. Obreimoff [5] are a priori and not natural boundary conditions and can be replaced by others if necessary. To the point, their applicability is usually related with fixing of a beam in a substantially more rigid medium, which does not take place in the case of a crack. In addition, it has been shown repeatedly [1, 4, 6] that in the vicinity of the crack tip, plastic flow occurs which developed to a greater or lesser degree depending on how long the crack remained in place. It is shown in [1] that with sufficiently slow deformation the sides of the crack receive considerable residual displacements due to bending of the crystal near the support. Even when fracture outwardly looks completely like brittle fracture, the size of the region at the crack tip covered by plastic flow is commensurable with the transverse dimensions of the beam. The hypothesis of a hinge in the fixed support is intended for taking these facts into consideration qualitatively without a detailed analysis of deformation in the vicinity of the crack tip.

We will derive the conditions of equilibrium of a crack with consideration that turning of the transverse section is permitted in the support. Let the crack lie in the midplane of the specimen so that we can consider only equilibrium of one beam. We denote by u (x) the displacement of its neutral axis for $0 \leq x \leq l$. For x = 0 the bending moment is absent, and displacement u = h. For $x = l$ we have zero displacement: $u(l) = 0$. The potential energy of the system being considered is written in the form

$$\frac{EI}{2}\int_0^l \left(\frac{d^2u}{dx^2}\right)^2 dx + \frac{Bb}{2}\left(\frac{du}{dx}\right)^2_{x=l} + Tbl, \qquad I = \frac{bH^3}{12} \tag{3.1}$$

In (3.1) the first addend is equal to the bending energy of the beam, the second is equal to the energy stored in the hinge, where B > 0 is rigidity of a hinge of unit width, and the third addend is the surface energy of one side of the crack. From the condition of the extremum of expression (3.1) we obtain, by calculus of variations methods [7], the mathematical statement of the problem (the function u (x) and crack length l are subject to variation):

$$d^4u \,/\, dx^4 = 0 \qquad (0 < x < l) \tag{3.2}$$

$$u = h, \quad d^2u \,/\, dx^2 = 0 \quad (x = 0) \tag{3.3}$$

$$u = 0, \quad (EI \,/\, Bb)\,(d^2u \,/\, dx^2) = -\,\partial u \,/\, dx, \quad x = l \tag{3.4}$$

$$(-EId^3 u \,/\, dx^3)\,(du \,/\, dx) + EI\,(d^2u \,/\, dx^2)^2 = 2Tb, \quad x = l \tag{3.5}$$

The second equation of (3.4) represents the equation of state of the hinge in the support (the angle of turn is proportional to the bending moment EId^2u/dx^2), and as $B \to \infty$ it passes to the condition of rigid fixing $du/dx = 0$. With such passage to the limit the second term in (3.1) disappears, since Bb du/dx tends to a finite value equal to the bending moment in the support. Condition (3.5) expresses that the work of the generalized forces during possible propagation of the crack tip is equal to zero. The first addend represents the work of the shearing force EId^3u/dx^3 spent on displacement caused by turning in the support. As $B \to \infty$ the angle of turn approaches zero according to (3.4), and the first term in (3.5) disappears. The second addend is equal to the work of the bending moment, and the right-hand side of (3.5) is equal to the linear density of the surface energy.

Solving problem (3.2)-(3.5), we obtain for the crack length the equation

$$\frac{9h^2}{l^4}\frac{1 + EI\,/\,Bbl}{(1 + 3EI\,/\,Bbl)^2} = \frac{2Tb}{EI} \tag{3.6}$$

We compare (3.6) with (2.1), which for $H_1 = H_2 = H$ acquires the form $9h^2/l^4 = 2Tb/EI$. To obtain a formal similarity of these formulas, we introduce the notation

$$T_+ = \frac{T(1+3EI/Bbl)^2}{1+EI/Bbl} \qquad (3.7)$$

If we assume the presence of rigid fixing at the crack tip and attempt to determine the density of the surface energy by Eq. (2.1) from experiments with specimens for which in reality there is elastic fixing (3.4) at the crack tip, the value not of T but of T_+ from (3.7) would be determined. As follows from (3.7), the quantity $T_+ > T$, and it depends on the crack length: $\lim T_+ = \infty$ for $EI/Bbl \to \infty$, $\lim T_+ = T$ for $EI/Bbl \to 0$.

Now in place of the first condition of (3.3) for $x = 0$ let the shearing force be given, i.e., $d^3u(0)/dx^3 = mg/EI$. Then to determine l we obtain in place of (2.2) the equation

$$\frac{6(mgl)^2}{Eb^2H^3}(1 + EI/Bbl) = T \qquad (3.8)$$

Equations (3.8) and (2.2) will agree formally if we set

$$T_- = T/(1 + EI/Bbl) \qquad (3.9)$$

In this case $T_- < T$, and it also depends on the crack length:

$$\lim T_- = 0 \quad \text{for} \quad EI/Bbl \to \infty \qquad \lim T_- = T \quad \text{for} \quad EI/Bbl \to 0$$

Consequently, if we assume the presence of elastic fixing at the crack tip, it becomes clear that in constructing distributions 1 and 2 in Fig. 3 the values of T_+ and T_- were calculated, respectively, as a consequence of which distribution 2 proved to be shifted along the x axis by two orders toward the smaller side in comparison with distribution 1.

The dimensionless parameter EI/Bbl is a measure of the ratio of the bending moment of the beam to rigid fixing. In particular, as $l \to \infty$ the beam becomes an ever more yielding system, and fixing can be regarded as rigid with ever greater validity. In conformity with this the values of T_+ and T_- approach T as $l \to \infty$.

This model of fixing describes phenomenologically the result of complex processes occurring in the vicinity of the crack tip under the effect of very high stresses. A detailed discussion of these phenomena is beyond the scope of beam theory and, possibly, beyond the scope of linear elasticity theory.

The matter is complicated even more by the fact that after the fracture condition (3.5) is attained and the crack begins to propagate, the high stresses, which gave rise to an additional store of energy at the point where there was fixing before the start of propagation, disappear, advancing along with the end of the crack. Therefore, it is necessary to make some assumptions concerning the fate of the energy stored in the fixed support before unloading. As microscopic observations show, at least a part of this energy remains as energy of dislocation loops in the crystal [1, 4, 6]. It was shown in [1] that this part is more appreciable, the slower the loading rate, so that at slow loading rates the hinge in the support is almost rigid-plastic.

With an increase of the loading rate the behavior of the hinge becomes more elastic. Since considerably fewer dislocations are produced during propagation of the crack than when it is stationary, it is natural to assume that the condition of rigid fixing is fulfilled during crack propagation. This assumption explains the stepwise propagation of a crack and the almost complete independence of the crack velocity from the loading conditions (it is equal to several tens of m/sec). With this interpretation of the facts the role of the external load amounts mainly to "charging" the hinge in the support with energy until condition (3.5) is attained. Propagation of the crack occurring thereafter is determined by the process of "discharging" of this hinge. Apparently on the basis of this approach we can explain the stepwise propagation of cracks as was done in [8]. In this connection, reference to [6] was erroneously made in [1]. The author offers his apologies to G. I. Barenblatt and R. L. Salganik who proposed this method. Although the considerations expressed are rather speculative, the circumstance that they explain the experimental facts gives them the right to exist. The drop of the effective surface energy upon an increase of crack length, as is predicted by Eq. (3.7), which was described earlier (see [1], Fig. 6), indicates in their behalf. This phenomenon was not explained in [1].

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